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ON COMPARING PRECISION ORBIT SOLUTIONS OF GEODETIC SATELLITES GIVEN SEVERAL ATMOSPHERIC DENSITY MODELS

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Many aspects of a satellite mission are directly impacted by the ability to precisely determine and accurately predict the satellite's orbit through high precision orbit determination. While gravity forces are typically well understood, the modeling of non-conservative forces to a high precision, which is critical to high precision orbit determination of satellites in low Earth orbit, is often more challenging. A number of current and historically recommended atmospheric density models are examined using the Naval Research Laboratory's Orbit Covariance Estimation and ANalysis (OCEAN) tool. High precision laser ranging data to geodetic satellites were used as test cases to evaluate the solution accuracy and predictive capabilities of the atmospheric density models. Orbit fit and prediction comparison metrics are generated for multiple atmospheric density models. Generally, the Jacchia-Bowman 2008 model results in predictive orbit solutions that more closely follow the definitive orbit solution over the entire 30 day prediction span. Surprisingly, the exponential atmospheric density model, while the simplest model, preforms almost as well over the first ten days of orbit prediction.

INTRODUCTION

Many aspects of a satellite mission are directly impacted by the ability to precisely determine and accurately predict the satellite's orbit through high precision orbit determination. High precision orbit determination requires precisely modeling all the forces imparted to satellite, including gravitational forces as well as a range of non-conservative forces. For satellites in Low Earth Orbit (LEO), atmospheric drag forces typically are the largest source of force modeling error. This is largely because the atmospheric density, to which drag forces are proportional, is difficult to characterize at satellite altitudes. A number of atmospheric density models have been used historically, or are currently recommended for orbit determination applications. The Naval Research Laboratory (NRL) Orbit Covariance Estimation and ANalysis (OCEAN) tool is used to evaluate the suitability of these models to the application of orbit determination. By characterizing the expected and relative performance of these models, recommendations for orbit determination processes may be made.

OCEAN is a highly configurable, database driven software tool that enables precision orbit determination for a range of satellite missions. OCEAN allows users to simulate data, propagate a spacecraft state, or solve for an orbit using a Kalman Filter-Smoother (KFS) or Weighted Least

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14. ABSTRACT
Many aspects of a satellite mission are directly impacted by the ability to precisely determine and accurately predict the satellite's orbit through high precision orbit determination. While gravity forces are typically well understood, the modeling of non-conservative forces to a high precision, which is critical to high precision orbit determination of satellites in low Earth orbit, is often more challenging. A number of current and historically recommended atmospheric density models are examined using the Naval Research Laboratory's Orbit Covariance Estimation and ANalysis (OCEAN) tool. High precision laser ranging data to geodetic satellites were used as test cases to evaluate the solution accuracy and predictive capabilities of the atmospheric density models. Orbit fit and prediction comparison metrics are generated for multiple atmospheric density models. Generally, the Jacchia-Bowman 2008 model results in predictive orbit solutions that more closely follow the definitive orbit solution over the entire 30 day prediction span. Surprisingly, the exponential atmospheric density model, while the simplest model, preforms almost as well over the first ten days of orbit prediction.

15. SUBJECT TERMS
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Squares Orbit Determination (WLS-OD) process. Early history of OCEAN is given in Reference 1, while references 2, 3, 4, 5, 6, and 7 discuss further developments. More recently OCEAN has been used to calculate orbits to support operations for the NRL UPPERSTAGE and TACSAT-4 satellite missions.

There has been much prior work in studying the relationship between atmospheric drag and satellite orbit determination. General studies of the accuracy of satellite drag modeling are found in References 8, 9, 10, and 11. The relationship between atmospheric drag modeling and the orbits of several geodetic satellites is examined in Reference 12.

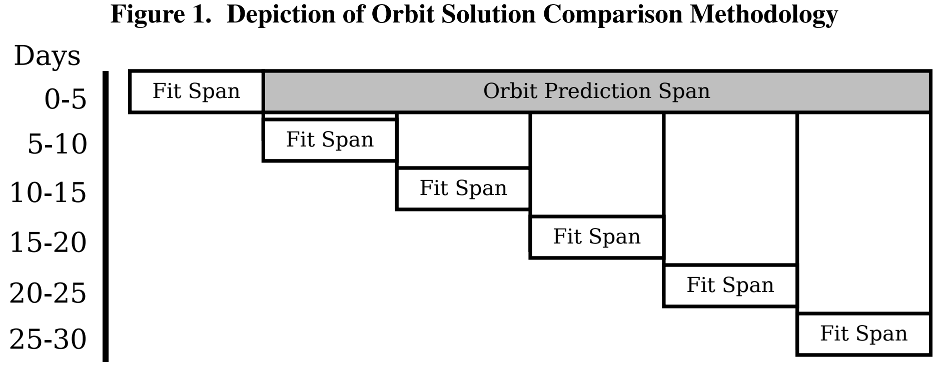
The primary focus here is to use high precision measurements to geodetic satellites to better understand how each atmospheric drag model impacts the ability to accurately predict an orbit.

ORBIT DETERMINATION METHODOLOGY

Many geodetic research activities are enabled by the global satellite laser ranging data coordinated and archived by the International Laser Ranging Service (ILRS).[13] A number of geodetic satellites have been designed and launched specifically to facilitate the study of Earth's gravity by incorporating laser retro-reflects to enable Satellite Laser Ranging (SLR) measurements. This SLR data for a number of satellites is archived by the ILRS. This high precision laser ranging data is well suited to perform precision orbit determination due to its low noise profile. ILRS guidelines call for a precision of the normal point laser range measurement to the geodetic satellite LAGEOS-1 of under one centimeter.[14]

OCEAN was used with SLR data to calculate precision orbits the geodetic satellites LARETS, STARLETTE and STELLA. By comparing predictive orbits to fitted orbits for each satellite using a several atmospheric density models, the predictive accuracy of the underlying models was evaluated. Orbits were determined with the OCEAN WLS-OD methodology using successive five day increments of SLR data. The RMS residual error was tabulated to better inform the quality of the orbit solution. The primary metric used was the comparison between an orbit prediction and subsequent orbit fit solutions. The orbit solution from the first five day data arc was propagated forward in time thirty days. The WLS-OD process was repeated for successive five day data arcs. These orbit solutions were compared to the predicted orbit from the first data arc solution. Thirty days was chosen as a comparison time span to demonstrate the longer term variation in predictive accuracy. The degree to which the orbit solutions agreed was used as a metric to evaluate the suitability of the atmospheric density models to precision orbit determination. This methodology is depicted in Figure 1.

Figure 1. Depiction of Orbit Solution Comparison Methodology



For each of these satellites, four atmospheric density models were evaluated. The data span was taken from 2012 since it is the most recent year average solar activity was observed. This was chosen to capture the average behavior of these models. Several satellites with differing inclinations and altitudes were used to better understand the impact the specific orbit has on the results. This methodology allows the predictive accuracy of each dynamic model to be evaluated as well as the quality of the orbit solution. The predictive accuracy of the orbit determination process is often of primary importance in satellite flight operations.

Details of each satellite used in the test cases are given.

STARLETTE

The Centre Nationale d'Etudes Saptiales (CNES) launched STARLETTE in 1975. It was designed to provide improved knowledge of the Earth's geopotential and to study solid Earth tides, ocean tides and polar motion.[15] STARLETTE was the first satellite to be covered entirely by laser retroreflectors, enabling passive SLR measurements.[16] This satellite's passive operations, known cross-sectional area, and low altitude cause it to be well suited for atmospheric drag studies.

The nominal orbital elements for STARLETTE are given in Table 1.

Table 1. Nominal Orbital Elements for STARLETTE

Element	Nominal Value
Semi-major Axis	$7,190$ km
Eccentricity	0.0206
Inclination	49.83^o

STELLA

CNES launched STARLETTE in 1993 as an essentially identical follow-on to STARLETTE. This satellite was placed in a higher inclination orbit to better capture Earth's gravitational variations at different latitudes.[17] STELLA is similarly well suited to study atmospheric drag effects.

The nominal orbital elements for the STELLA satellite are given in Table 2.

Table 2. Nominal Orbital Elements for STELLA

Element	Nominal Value
Semi-major Axis	$7,178$ km
Eccentricity	0.0206
Inclination	98.6^o

LARETS

LARETS was launched by the Russian Space Agency (RSA) in 2003 as a follow on the WESTPAC satellite, which launched six years earlier.[18] LARETS was launched into a lower altitude orbit to better study geodynamics. Likewise, this satellite's passive operations, known cross-sectional area, and low altitude cause it to be well suited for atmospheric drag studies[19].[20]

The nominal orbital elements for the LARETS satellite are given in Table 3.

Table 3. Nominal Orbital Elements for LARETS

Element	**Nominal Value**
Semi-major Axis	$7,068$ km
Eccentricity	0.0023
Inclination	97.7^o

DRAG MODELS

A number of atmospheric density models have been used for calculating drag on satellites both historically and according to current best practices. The exponential atmospheric model, the Jacchia 1970 model, the Naval Research Laboratory Mass Spectrometer and Incoherent Scatter Radar 2000 (NRLMSISE-00) model, and the Jacchia-Bowman 2008 model were examined as they are a representative set of atmospheric density models.

The exponential atmospheric density model is the simplest model examined. It models atmospheric density as an exponential function and does not account for diurnal, semi-diurnal, or solar cyclical variations, or the variations due to rotating atmospheric effects. Additionally, this model does not use solar indices as input data.[21]

The Jacchia 1970 model has been frequently used in astrodynamics.[21] The model contains analytical expressions to calculate densities based off of observed solar and geomagnetic activity captured by solar indices. The model also includes diurnal and semi-diurnal effects, as well as effects due to the rotating atmosphere.[22] This is one of several atmospheric density models constructed throughout the long career Dr. Luigi G. Jacchia. More information on these series of models are in References 23, 22, and 24.

The NRLMSISE-00 atmospheric semi-empirical model calculates temperatures and densities for the atmospheres constituent components. This model is based upon mass spectrometer and incoherent scatter data from a linage of a number of models, such as those discussed in References 25,26, 27, and 28. The model was developed to improve upon the Jacchia 1970 as well as the MSISE-90 models by incorporating data from observed satellite orbits and on orbit accelerometer data. This model includes diurnal and semi-diurnal effects and is driven by observed solar and geomagnetic activity indices.[29]

The Jacchia-Bowman 2008 atmospheric empirical model is based on earlier Jacchia models. It includes the diurnal and semi-diurnal effects and is also driven by observed solar and geomagnetic activity indices. However, this model makes use of measurements of the solar flux in the Ultraviolet spectrum, which is currently recognized as having an appreciable impact on atmospheric heating. The model also makes use of a geomagnetic storm index to better capture atmospheric behavior during these periods of high solar activity.[30] This model builds on the Jacchia-Bowman 2006 model and is related to the High Accuracy Satellite Drag Model[31].[32]

Each of these models are evaluated for data from several satellite test cases. For each case, the orbital position and velocity, as well as the coefficients of drag and solar radiation pressure are estimated. Station specific biases as well as empirical accelerations are not estimated. This allows the effects of the dynamical models to be evaluated, rather than the ability to estimate unmodeled error sources.

TESTING RESULTS

Separate results are presented for each satellite considered. For each satellite, Root Mean Square (RMS) residual errors are given for each fit span as a measure of orbit solution quality. While RMS residual error can indicate the quality of the orbit fit to the measurement data, it often does not provide guidance as to the predictive accuracy of the orbit solution.[5] Thus, Residual Sum of Squares (RSS) position errors between the initial interval orbit prediction and the current interval orbit solution are given to show the predictive accuracy of the orbit solution for a specific atmospheric density model. Each fit interval is five days long. Plots of average daily RSS error are given as a metric to evaluate the suitability of the underlying model for precision orbit determination.

STARLETTE Results

The results from the STARLETTE test cases are presented below.

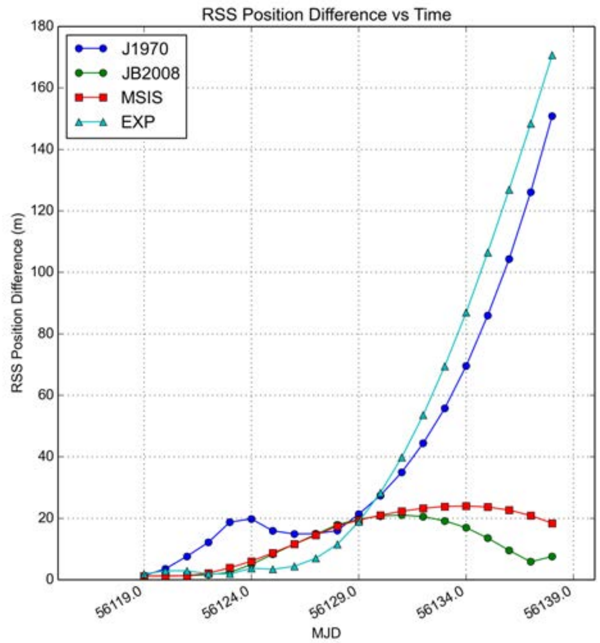
The RMS residual error for each fit span is given in Table 4. As can be seen, the RMS residual error is almost always higher when using the Jacchia 1970 (J1970) model. The lowest RMS residual error is seen when using the Jacchia-Bowman 2008 (JB2008) model. The NRLMSISE-00 model has only slightly higher RMS residual error. Surprisingly, the exponential model (Exp.) does not yield significantly worse RMS residual error.

The plot of average daily RSS position error between the predicted orbit and fit orbit for each atmospheric density model is given in Figure 2. As can been seen, both the JB2008 and NRLMSISE-00 models have orbit predictions that most closely match the fitted orbit. Surprisingly, the exponential model's predicted orbit most closely matched the fitted orbit in the second 5 day interval; however, it quickly diverges from the fitted orbit afterwards. The J1970 model's orbit prediction shows the least consistency of all the models.

Table 4. RMS Error Residuals in Meters for STARLETTE Test Cases

Fit Span	Exp. Model	J1970 Model	NRLMSISE-00 Model	JB2008 Model
Interval 1	0.616052	0.614464	0.416306	0.403346
Interval 2	0.569913	0.887697	0.559023	0.552870
Interval 3	0.461790	0.702634	0.398569	0.390309
Interval 4	0.351999	0.354915	0.343710	0.343069
Interval 5	0.573561	0.487481	0.485936	0.463770

Figure 2. STARLETTE Average Daily RSS Orbit Difference Between Predicted Orbit and Fit Orbit For Various Atmospheric Density Models



STELLA Results

The results from the STELLA test cases are presented below.

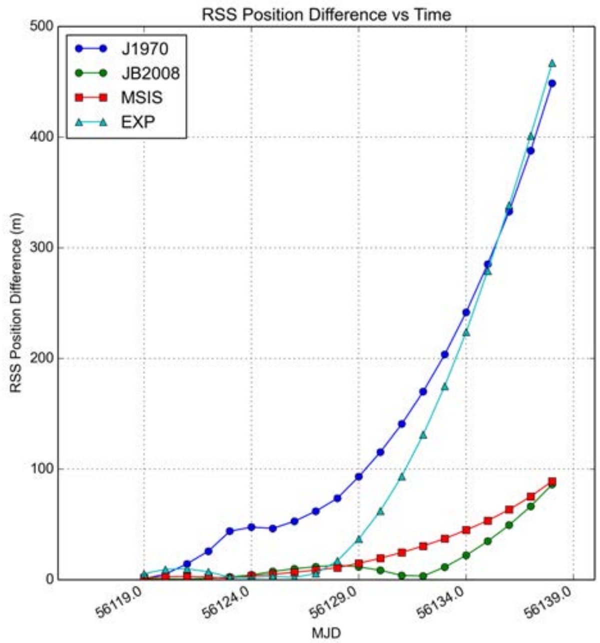
The RMS residual error for each fit span is given in Table 5. As can be seen, the RMS residual error is almost always higher when using the Jacchia 1970 (J1970) model. The lowest RMS residual error is seen when using the Jacchia-Bowman 2008 (JB2008) model or the NRLMSISE-00 model. Surprisingly, the exponential model (Exp.) does not yield significantly worse RMS residual error in most cases.

The plot of average daily RSS position error between the predicted orbit and fit orbit for each atmospheric density model is given in Figure 3. As can been seen, both the JB2008 and NRLMSISE-00 models have orbit predictions that most closely match the fitted orbit. Surprisingly, the exponential model's predicted orbit again shows close consistency within the first ten days; however, it quickly diverges from the fitted orbit afterwards. The J1970 model's orbit prediction shows the least consistency of all the models.

Table 5. RMS Error Residuals in Meters for STELLA Test Cases

Fit Span	Exp. Model	J1970 Model	NRLMSISE-00 Model	JB2008 Model
Interval 1	1.043630	1.349570	0.650235	0.593723
Interval 2	0.679715	0.966544	0.371760	0.397893
Interval 3	0.503609	1.010750	0.424553	0.447889
Interval 4	0.376582	0.322984	0.308842	0.303791
Interval 5	0.799102	0.527687	0.527811	0.363473

Figure 3. STELLA Average Daily RSS Orbit Difference Between Predicted Orbit and Fit Orbit For Various Atmospheric Density Models



LARETS Results

The results from the LARETS test cases are presented below.

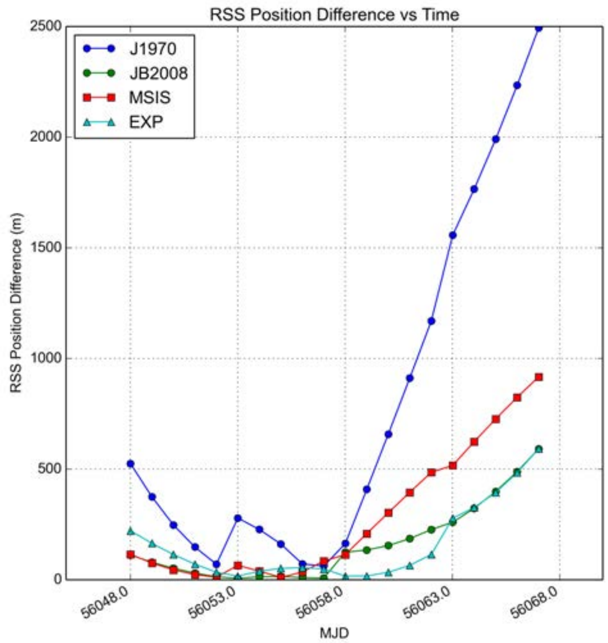
The RMS residual error for each fit span is given in Table 6. As can be seen, the RMS residual error is almost always higher when using the Jacchia 1970 (J1970) model. The lowest RMS residual error is seen when using the Jacchia-Bowman 2008 (JB2008) model, signficantly so in interval five. The other models show a wide range of variability. This could be due to the lower altitude of LARETS, which would cause the difficult to model drag forces to be more dominant, as well as the poorer orbit fit quality in general.

The plot of average daily RSS position error between the predicted orbit and fit orbit for each atmospheric density model is given in Figure 4. Here, the JB2008 models shows the most consistent performance. The exponential model also performs well during most of the month, often more closely matching the predicted orbit than the other models. The J1970 model's orbit prediction shows the least consistency of all the models.

Table 6. RMS Error Residuals in Meters for LARETS Test Cases

Fit Span	Exp. Model	J1970 Model	NRLMSISE-00 Model	JB2008 Model
Interval 1	0.81661	1.09324	0.552407	0.600308
Interval 2	3.61143	3.51835	0.934227	0.750071
Interval 3	1.23148	3.77240	2.189620	1.453930
Interval 4	0.89638	1.24462	0.840829	1.150560
Interval 5	3.78744	4.89433	1.445030	0.845118

Figure 4. LARETS Average Daily RSS Orbit Difference Between Predicted Orbit and Fit Orbit For Various Atmospheric Density Models



CONCLUSION

Multiple atmospheric density models are examined in the orbit determination process for multiple geodetic satellites. While it is difficult to generalize these results from a limited number of test cases to make a recommendation for general use, it is apparent that the Jacchia-Bowman 2008 model yields orbit predictions that most consistently match the orbit fit. A surprising result is that the exponential atmospheric density model often performs quite well against more complex models. The Jacchia 1970 model consistently shows the poorest matching between the predicted and fit orbits. The use of the Jacchia 1970 model also correlates to the poorest RMS residual errors. This indicates the orbit solution quality during the fit span is worse when using this model.

While this work is encouraging, more analysis is needed to make a more general recommendation as to the suitability of these models in applications where predictive orbit accuracy is critical. For example, these data were analyzed during average solar conditions. This analysis should be repeated during solar maximum and solar minimum to better inform each model's ability to capture the atmosphere's behavior during solar extrema. Last, a more thorough analysis using more data from more satellite can better inform the expected performance.

APPENDIX: EXAMINATION OF ORBIT PREDICTION METRICS

It has been suggested that comparing solution orbits to the reference ephemeris generated by one of the ILRS analysis center is a more relevant metric than comparing orbit predictions to orbit fit solutions. These reference ephemerides, however, are generated using estimated site biases, empirical accelerations and spacecraft biases. The resulting orbit solution may include deviations due to estimation error of these parameters rather than to physical forces. Ultimately, the orbit

solution that is treated as truth largely does not matter, as the differences between these orbits are orders of magnitude smaller than the differences between the predictive orbit and the fit orbit solution.

As an example of this, the differences between the orbit solutions generated during the fit span for each test case is plotted. Here, the JB2008 model solution is used as an orbit truth and the average daily RSS position difference for the other orbit solutions are plotted.

As can be seen in Figure 5, the difference between any of the orbit solutions is between ten centimeters and one meter for the Starlette test cases. This is at least two orders of magnitude less than the difference between the predicted orbit and fit orbit solutions.

Figure 6 shows the differences for the Stella test cases. Similar behavior is true for these test cases.

Figure 7 shows the differences for the Larets test cases. Interestingly, the orbit differences are much greater for this test cases. However, the orbit differences are still at least two orders of magnitude less than the difference between the predicted and fit orbits for the vast majority of the time span examined.

Thus, the choice of ephemeris to regard as truth makes no appreciable difference in this analysis.

Figure 5. Starlette Average Daily RSS Orbit Difference Between JB2008 Fit Orbit and Other Atmospheric Density Model Fit Orbits

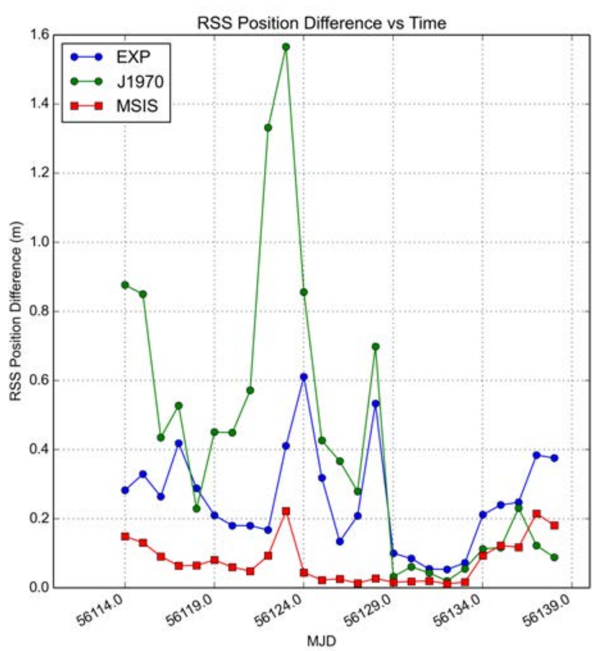


Figure 6. Stella Average Daily RSS Orbit Difference Between JB2008 Fit Orbit and Other Atmospheric Density Model Fit Orbits

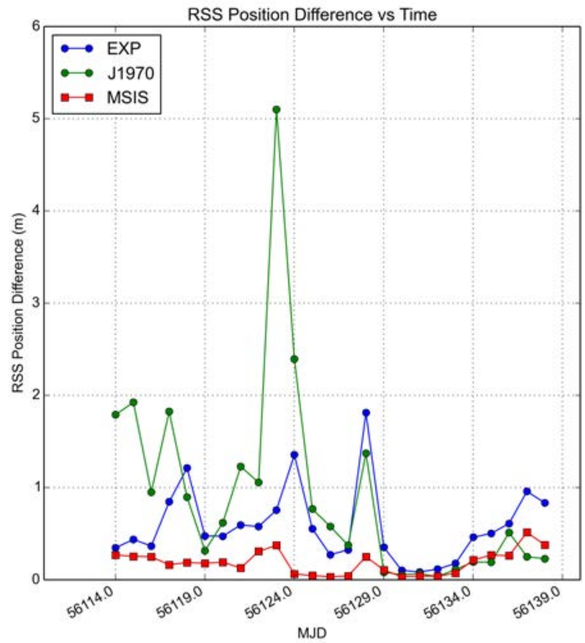
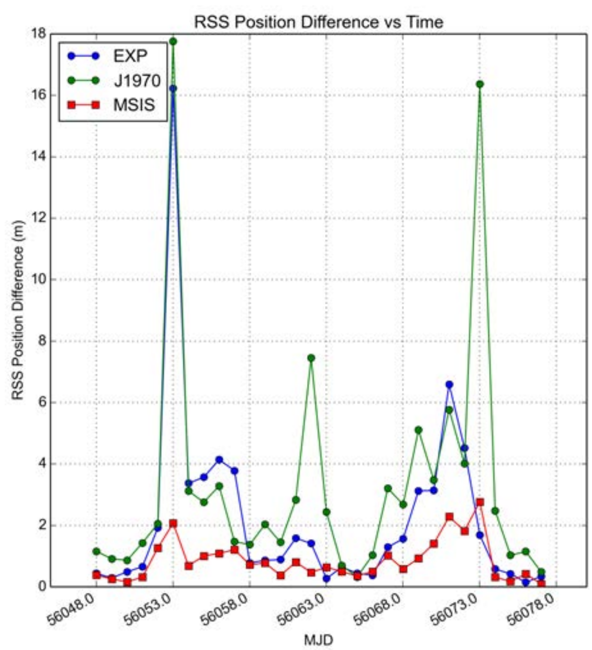


Figure 7. Larets Average Daily RSS Orbit Difference Between JB2008 Fit Orbit and Other Atmospheric Density Model Fit Orbits



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